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Computation of input impedance of rectangular, circular and hexagonal patch microstrip antennas in *S* and *X*-band

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Abstract Matching of input impedance of an antenna with its feed network is one of the determining factor for its best performance. In this paper, we have presented improved formulae for the input impedance of three geometries namely rectangular, circular and hexagonal patch antennas. The input impedance for these geometries has been computed and plotted as a function of feed point location in *S* (3 GHz) and *X* (10 GHz) band of microwave frequency range. Some other important antenna parameters like quality factor, directivity, efficiency, gain, bandwidth, beamwidth, radiation resistance *etc*, have also been computed and presented. A comparision of these antenna characteristics has also been made for two operating frequencies in *S* and *X*-band respectively.

Keywords Microstrip antennas, input impedance, antenna parameters.

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1. Introduction

Microstrip antennas are used in broad range of applications from communication systems (radars, navigation and telemetry) to biomedical systems, primarily due to their simplicity, conformability, low manufacturing cost and enormous availability of design and analysis software [1–7]. The radiating element of an antenna and the feed line are usually photoetched on the dielectric substrate. Normally, printed dipole is used either in its half wave length or full wave length form as a radiating element. However, the patch design has much greater flexibility in that many shapes of patch can be used to realize specific characteristics.

Without adequate attention to feed, the actual antenna cannot function properly, despite the radiating element being designed with care and precision. Since insertion loss of the feed increases drastically with frequency hence additional care is required for feed design at higher frequencies. Matching may be achieved by properly selecting the location of the feed.

2. Formulation and computation of the problem

Rectangular patch microstrip antenna (RPMA) :

The RPMA is modelled as a cavity with a magnetic wall along the edge and electric walls above and below. The fields in the antenna are assumed to be those of the cavity. Hence the input impedance at any feed point may be evaluated. The geometry and coordinate system of RPMA is shown in Figure 1.

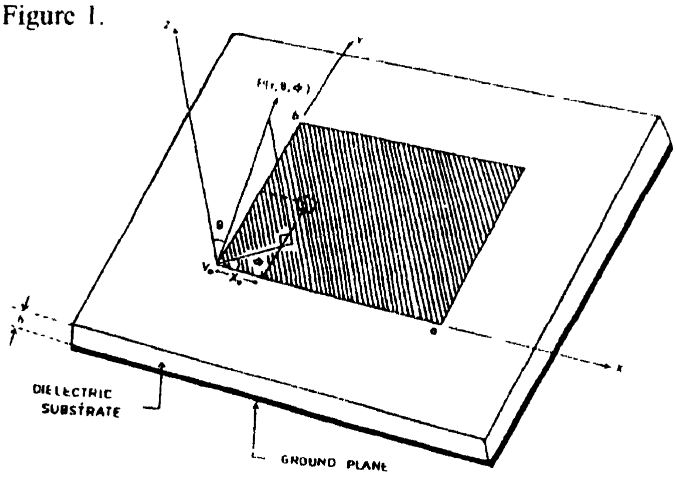


Figure 1. Geometry and coordinate system of RPMA

In RPMA the modes of interest are TM mode, which have $H_z = 0$ but a non-zero value of E_z. In this mode the field components are given by [8]

$$E_z = C_{nm} \cos[(n\pi / a)x] \cos[(m\pi / b)y], \quad (1)$$

$$H_x = -\frac{jC_{nm}}{k_0 Z_0} \frac{m\pi}{b} \cos\left[\frac{n\pi}{a} x\right] \sin\left[\frac{m\pi}{b} y\right], \quad (2)$$

$$H_y = \frac{jC_{nm}}{k_0 Z_0} \frac{m\pi}{a} \sin\left[\frac{n\pi}{a} x\right] \cos\left[\frac{m\pi}{b} y\right] \quad (3)$$

where n and m are integers and $n = m = 0$ excepted C_{nm} is amplitude constant and for loss free conditions may be written as

$$C_{nm} = \frac{\varepsilon_{0n}\varepsilon_{0m}}{ab}\int_0^a\int_0^b \frac{jk_0 Z_0 J_z(x,y)\cos\left(\frac{n\pi}{a}x\right)\cos\left(\frac{m\pi}{b}y\right)}{k^2 - k^2_{nm}}\,dx\,dy, \tag{4}$$

$$\varepsilon_{0n}, \varepsilon_{0m} = \begin{cases} 1 \text{ for } n = m = 0 \\ 2 \text{ for } n, m > 0, \end{cases} \tag{5}$$

$$k^2 = \omega^2\mu_0\varepsilon_0 \tag{6}$$

k_{nm} is resonant wave number of the nm-th mode defined as

$$k^2_{nm} = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 \tag{7}$$

For RPMA the desired mode is TM_{10} so if we choose $k \approx k_{10}$ then this mode is excited with a large amplitude. Thus using eq. (5) and taking the dielectric losses ($i.e.$, $k^2 = \omega^2\mu_0(\varepsilon' - j\varepsilon'')$), conductor losses, radiation losses along with the effect of energy storage outside the cavity (by modelling the cavity walls as walls having a finite complex admittance instead of zero admittance) into account [8–10], the expression for C_{10} becomes

$$C_{10} = -\frac{2k_0 Z_0 Q_{10}\varepsilon' I_0 \cos\left(\frac{\pi x_0}{a}\right)}{k_{10}^2 ab\varepsilon} \tag{8}$$

where I_0 is the total current in the probe of radius r_0

From the defination of quality factor Q_T, we have

$$P_T = \omega\frac{W_T}{Q_T}, \tag{9}$$

where P_T = total power dissipated,

W_T = total stored energy, obtained as follows,

$$W_T = \frac{\varepsilon'}{2}\int_v |E_z|^2\,dv \tag{10}$$

We can define an input resistantce as seen from the coaxial line, by the relationship $1/2\,|I_0|^2 R_{in} = P_T$. Hence at resonance, R_{in} is

$$R_{in} = \frac{2\omega\varepsilon' h|k_0 Z_0\varepsilon'\cos(\pi x_0/a)|^2 Q_r}{k_{10}^2 ab} \tag{11}$$

Thus by choosing x_0 appropriately, the cavity can be matched to the input coaxial line. The input impedance Z_a may be given as

$$Z_a = -\frac{2jZ_0 h\cos^2(\pi x_0/a)}{ab\varepsilon_r k_0}\,\frac{\omega^2}{\left[\omega^2 - \omega_{10}^2(1 + j/2Q_T)\right]} \tag{12}$$

All of the non-resonant modes combine to give the inductive self reactance of the probe, so that the final expression for Z_a is

$$Z_a = \frac{j\omega\mu_0 h}{2\pi}\ln\sqrt{\frac{ab}{\pi r_0^2}} - \frac{2jZ_0 h\cos^2(\pi x_0/a)}{\varepsilon_r k_0 ab}\,\frac{\omega^2}{\omega^2 - \omega_{10}^2(1 + j/2Q_T)^2}, \tag{13}$$

where a and b are the length and width of the rectangular patch respectively. Antenna quality factor Q_T is associated with the system losses, including radiation losses Q_r, dielectric losses Q_d and conductor losses Q_c. The expression for quality factor may be given as follows :

$$\frac{1}{Q_T} = \left(\frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_r}\right), \tag{14}$$

where $Q_d = \frac{1}{\tan\delta}$, $Q_c = \frac{\omega\varepsilon' h\sigma\delta_s(k_0 Z_0)^2}{2k_{nm}^2}$ and $Q_r = \frac{k_x'}{2k_x''}$

Here, $\delta = \left(\frac{2}{\omega\mu\sigma}\right)^{1/2}$ is the skin depth,

σ = conductivity of metalization,

$$k_x = \pi/a + \frac{2\alpha_x\pi}{\pi^2 - (a\alpha_x)^2},$$

$$\alpha_x = j\,k_0\,Z_0\,Y'_{wx}\,F_x,$$

$Y'_{wx} = 0.00836\frac{h}{\lambda_0} + j\,0.01668\frac{\Delta l}{\lambda_0}\varepsilon_e$ is wall admittance along x direction

and $F_x = 0.7747 - 0.5977\,(1 - b/a) - 0.1638\,(1 - b/a)^2$

The length extention (Δl) and the effective permittivity (ε_e) are obtained following Refs. [8,9].

The input impedance of RPMA has been computed for two cases taking source frequency 3 GHz (S-band) and 10 GHz (X-band) respectively. The variation of impedance with position of feed point is shown in Figure 2.

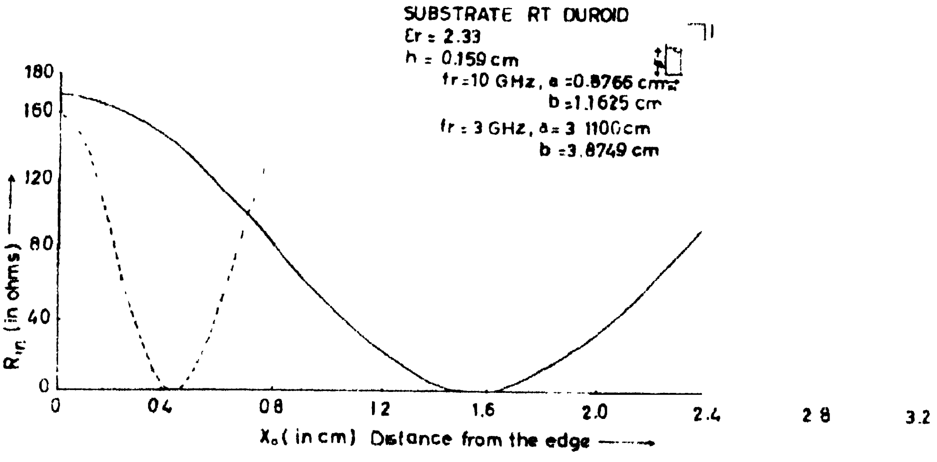


Figure 2. Variation of input impedance as a function of feed point location for RPMA.

It is evident from the figure that the input inpendance matches with the inset coaxial feed (50 ohms) at two feed point locations *i e.*, x = 0.9898 cm, y = 1.9375 cm and x – 2 1213 cm, y = 1.9375 cm for 3 GHz and x = 0.2723 cm, y – 0 5812 cm and x – 0.6043 cm, y = 0.5812 cm for 10 GHz.

3. Antenna parameters of RPMA

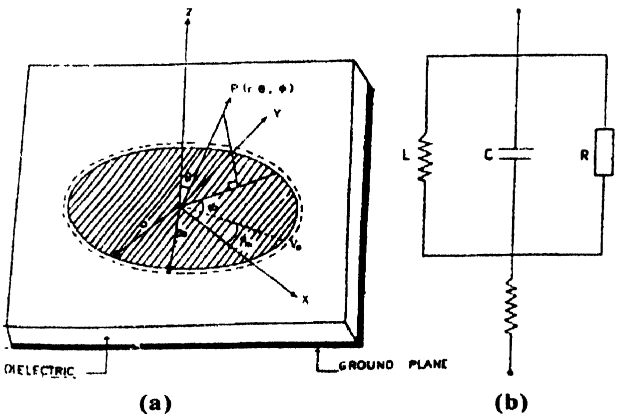
The important antenna parameters of RPMA including radiation resistance, equivalent resistance for the copper loss R_c and dielectric loss R_d, antenna efficiency (η), bandwidth (B W), directivity (D_w) and gain have been computed in S and X-band and presented in Table 1.

Table 1. Antenna parameters of RPMA.

Sl Nos	Antenna parameters	Design frequency (in GHz)	
		3 (S-Band)	10 (X-band)
1	a (in cm)	3 1111	0.8766
2	b (in cm)	3.8749	1.165
3	ε_r	2.2225	2.0972
4	Q_T	25 1783	6 6125
5	Z_{in}	$[171\ 0129 - j\ 1\ 698] \cos^2\left(\frac{\pi x_0}{a}\right)$	$[159.1829 - j\ 6\ 018] \cos^2\left(\frac{\pi x_0}{a}\right)$
6	R_{in} (in ohms)	$171\ 0129 \cos^2\left(\frac{\pi x_0}{a}\right)$	$159.1829 \cos^2\left(\frac{\pi x_0}{a}\right)$
7	R_c (in ohms)	0 2380	0.02815
8	R_d (in ohms)	0 71053	0.1739
9	R_r (in ohms)	599 39	599 4
10	R_T (in ohms)	300 648	299 9
11	η (in %)	99 68	99 93
12	D_w (in dB)	7 78	7 78
13	Gain	7 75	7.8
14	BW (in %)	1 62	6 17

Circular patch microstrip antenna (CPMA) :

The geometry and coordinate system of CPMA is shown in Figure 3(a).



(a) **(b)**

Figure 3. (a) Geometry and coordinate system of CPMA . and (b) Equivalent resonant parallel L-C-R circuit.

The far-zone fields of this geometry obtained using cavity model [8] are as follows :

$$E_\phi = j^n \frac{Vak_0}{2r} e^{-jk_0 r} \cos\theta \sin n\phi$$

$$[J_{n+1}(k_0 a \sin\theta) + J_{n-1}(k_0 a \sin\theta)], \quad (15)$$

$$E_\theta = j^n \frac{Vak_0}{2r} e^{-jk_0 r} \cos n\phi$$

$$[J_{n+1}(k_0 a \sin\theta) - J_{n-1}(k_0 a \sin\theta)], \quad (16)$$

where E_ϕ, E_θ = components of total electric field vector for EM wave,

J_{n+1}, J_{n-1} – (n + 1)-th and (n – 1)-th order Bessel's functions of first kind respectively.

$V = hE_0 J_n\,(ka)$ is the edge voltage at ϕ = 0.

At resonance, the input impedance of a microstrip antenna is real. If the disk is fed at an arbitrary point (ρ_0,0,0), the resistance at resonance is

$$R = \frac{h^2 E_0^2 J_n^2\,(k\rho_0)}{2P_T}, \quad (17)$$

where $P_T = P_r + P_c + P_d$.

The radiated power P_r is written as follows [8] :

$$P_r = \frac{(hE_0 J_n\,(ka)\,ak_0)^2}{1920} I_1. \quad (18)$$

Here, $$I_1 = \int_0^\pi \left[J_{n+1}(k_0 a \sin\theta) + J_{n-1}(k_0 a \sin\theta)\right]^2$$

$$+ \cos^2\theta \left[J_{n+1}(k_0 a \sin\theta) + J_{n-1}(k_0 a \sin\theta)\right]^2 \sin\theta d\theta.$$

The power dissipated due to conductor loss P_c is given as

$$= \left(\frac{\pi f \mu}{\sigma}\right)^{1/2} \frac{E_0^2}{(\omega\mu)^2} \pi \left[\frac{1}{2} J_n^2(ka)\{(ka)^2 - n^2\}\right]. \quad (19)$$

The dielectric loss is determined by integrating the electric field inside the cavity over the volume V and may be given as follows :

$$P_d = \frac{h \tan\delta E_0^2}{8\mu f} J_n^2(ka)\left[(ka)^2 - n^2\right]. \quad (20)$$

From eq. (17), R may be calculated for any position of the source. The approximate value of input impedance can be determined by using a simple parallel LCR resonant circuit shown in Figure 3(b). At resonance the circuit may consist of resistance and inductance in parallel with each other *i.e.*,

$$Z = \frac{\omega^2 L^2 R + j\omega L R^2}{R^2 + \omega^2 L^2} = R_1 + jX_1, \quad (21)$$

$$Q_T = \frac{X_1}{R_1} = \frac{R}{\omega L}, \quad (22)$$

$$L = \frac{}{2\pi f_r Q_T}. \quad (23)$$

Alternatively, in the similar condition, the circuit may consist of resistance and capacitance in parallel with each other and therefore, the capacitance (C) is obtained

$$C = \frac{Q_l}{2\pi f_r R}. \tag{24}$$

Thus, knowing the values of R, L and C, the input impendance can be found from the relation

$$Z_{in} = R_{in} + jX_{in} = \left[\frac{1}{R} + j\omega C + \frac{1}{j\omega L}\right]^{-1} \tag{25}$$

(a) Input impedance at resonance :

At resonance the input impedance $Z_{in} = R_{in} + R$. (26)

(b) Input impedance at off resonance :

The quality factor Q_T necessary for calculating the input impedance at frequencies off the resonance is defined by eq (9) where the total stored energy W_l is given by [8] :

$$W_T = \frac{hE_0^2}{8\omega f\mu} J_n^2(ka)[(ka)^2 - n^2]. \tag{27}$$

Further, following eq. (9) and (27), expressions for Q_c, Q_d and Q_r can be obtained as

$$Q_c = h(\pi f\mu\sigma)^{1/2}, \tag{28}$$

$$Q_d = \frac{1}{\tan\delta}, \tag{29}$$

$$Q = \frac{240\left((ka)^2 - n^2\right)}{hf\mu k_0^2 a^2 I_1}. \tag{30}$$

Thus knowing the total quality factor Q_T, the input impedance at any frequency can be found from eq. (25).

The avove formula for input impedance can be improved by considering (i) the effect of wall admittance due to which the eigenvalue becomes complex and has a real value slightly less than 1.84118 and (ii) the effect of series inductive reactance due to probe. Therefore, considering these effects and using modal expansion model in the vicinity of dominant mode resonant frequency [8,9], the modified formula for the input impedance of CPMA may be written as follows :

$$Z_{in} = jX_L - j\frac{\omega / C}{\omega^2 - \omega_{11}^2(1 + j/Q_T)} J_1^2 \operatorname{Re}(k_{11}\rho_0), \tag{31}$$

where

X_l – series inductive reactance associated with the probe,

ρ_0 – feed point from the centre of the circular disk,

$C = \dfrac{\varepsilon_0 \varepsilon_r a^2}{2.775h}$ is the patch capacitance,

$k_{11}a = 1.84118 - \Delta_5 = \operatorname{Re}(k_{11}a) + j\operatorname{Im}(k_{11}a)$,

Δ_5 can be obtained from iterative algorithm defined as [8].

$$\Delta_{p+1} = \frac{1.8410969(1-\alpha a) + 4.0260952\Delta_p^2(1-\alpha a) - 1.84118}{3.3263839(1-\alpha a) - 1}$$

with ($\Delta_0 = 0$),

$$\alpha = j\frac{\eta_0 h}{\lambda_0 a}(G_w + jB_w).$$

Here, η_0 is the free space characteristic impedance and G_w and B_w are wall conductance and susceptance respectively and given as

$$G_w = 0.01254\frac{\pi a}{\lambda_0} \text{ mhos}, \tag{32}$$

$$B_w = 0.00834\frac{\pi a \varepsilon_r}{\lambda_0}. \tag{33}$$

The input impedance of CPMA has been computed at two frequencies *i.e.*, 3 GHz and 10 GHz. The variation of input impedance is plotted in Figure 4.

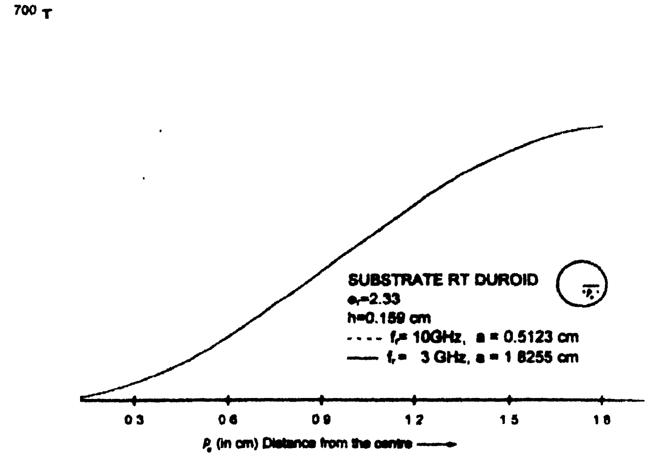


Figure 4. Variation of input impedance as a function of feed point location for CPMA

From figure it is observed that the input impedance of the CPMA matches with the inset coaxial feed *i e.*, 50 ohms at ρ_0 = 0.3847 cm for 3 GHz and ρ_0 = 0.0999 cm for 10 GHz.

4. Antenna parameters of CPMA

The important antenna parameters such as radius (a), effective radius (a_e), efficiency (η), bandwidth (BW), directivity (D), gain (G) *etc.* have been computed for CPMA in S and X-band and are presented in Table 2.

Table 2. Antenna parameters of CPMA

Sl Nos	Antenna parameters	Design frequency (in GHz)	
		3 (S-Band)	10 (X-band)
1	a (in cm)	1 8255	0 5123
2	a_e (in cm)	1 9241	0.5813
3	Resonant freq f_r (in GHz)	2.99	9 907
4	G_w (in mhos)	0 0072	0.0067
5.	B_w (in mhos)	0 0111	0.0104
6	Q_l	42.6776	14 6859
7	R_{in} (in ohms)	1455 128 $J_1^2(0.981\rho_0)$	1907.577 $J_1^2(3.282\rho_0)$
8.	R_r (in ohms)	455 79	4999.04
9	η (in %)	93 97	98.43
10	BW (in %)	0 9566	2.7799
11	D (in dB)	6 99	6 8
12	G (in dB)	6 7	6.7

Hexagonal patch microstrip antenna (HPMA)

The geometry and coordinate system of HPMA is shown in Figure 5.

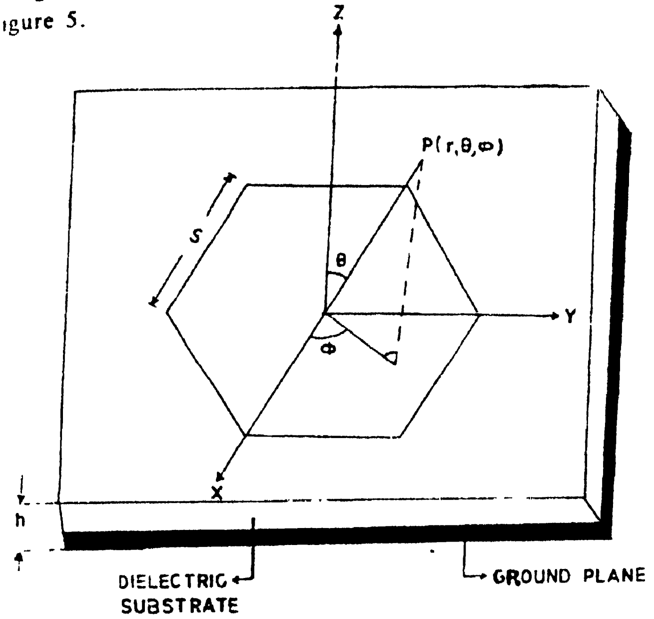


Figure 5. Geometry and coordinate system of HPMA

Because a circular patch is a limiting case of polygon with a large number of sides, the resonant frequency for the dominant as well as for the higher order modes may be calculated using the similar expression as for CPMA by replacing the radius a of circular patch by equivalent radius a_{eq}. The equivalent radius a_{eq} is determined by comparing areas of a hexagon and a circular disk of radius a_{eq}, i e

$$a_{eq} = 0.9094\ S,$$

where S is one side of hexagon as shown in Figure 5

Using the above concept and replacing the radius a of CPMA by a_{eq} in equations 31 to 33, an expression for input impedance of HPMA may be obtained. We have computed the values of input impedance for various values of feed point location for HPMA at 3 GHz and 10 GHz and the results are plotted in Figure 6.

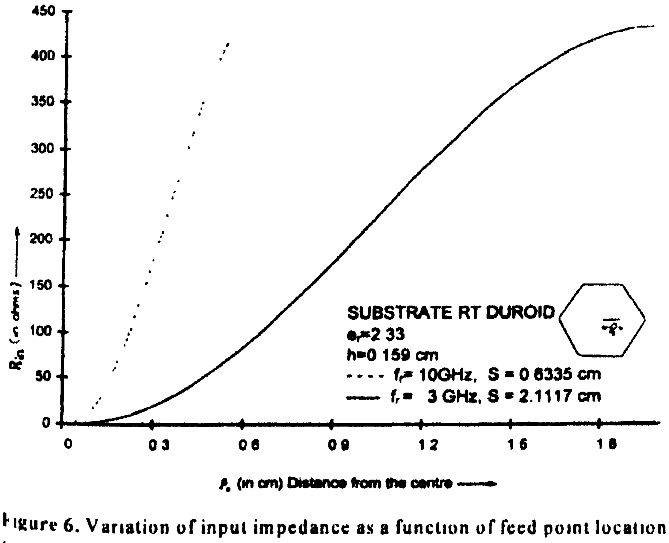


Figure 6. Variation of input impedance as a function of feed point location for HPMA

It is observed from the figure that the input impedance of HPMA matches with the impedance of inset coaxial feed (50 ohms) at ρ_0 = 0.4365 cm and 0.1362 cm for 3 and 10 GHz respectively.

Antenna parameters of HPMA

The important antenna parameters of HPMA have been estimated on both the frequencies i e , 3 GHz and 10 GHz and are summarised in Table 3.

Table 2. Antenna parameters of HPMA

Sl Nos	Antenna parameters	Design frequency (in GHz) 3 (*S*-Band)	10 (*X*-band)
	S (in cm)	2.1117	0.6335
	a_{eq} (in cm)	1.9204	0.5761
	Resonant freq f_r (in GHz)	2.9989	9.907
4	G_w (in mhos)	0.0076	0.0076
5	B_w (in mhos)	0.0117	0.0117
6	Q_T	41.4299	13.0203
7	R_{in} (in ohms)	$1276.43\ J_1^2(0.9256\rho_0)$	$1337.16\ J_1^2(2.895\rho_0)$
8	R_r (in ohms)	441.64	441.64
9	η (in %)	94.149	98.608
10	BW (in %)	0.985	3.135
11	D (in dB)	7.29	7.29
12	G (in dB)	7.028	7.229

5. Conclusion

Improved formulae for the input impedance of rectangular, circular and hexagonal antennas have been obtained These expressions are very much required in matching the antenna to the coaxial line. Expression for the input impedance of rectangular patch microstrip antenna has been developed by using cavity model, while for circular and hexagonal patch antenna, cavity model along with model expansion model and equivalent resonant parallel LCR circuit have been used The variation of input impedance at resonance with location of feed point for all the three geometries in the S and X-band are shown in Figures 2, 4 and 6 respectively Important antenna parameters of these geometries have also been computed and presented in Tables 1 3. For this analysis, all the three antenna geometries are designed on double sided copper claded RT duroid substrate (ε_r 2.33) of height, h = 0 159 cm and loss tangent, tan δ = 0.00066. Our results are capable to provide the exact position of feed points for these antenna geometries over to S and X-band for different inset coaxial feed line Overall, the results of the present investigation may be useful for a prospective antenna designer

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